Vacuum effects in electrodynamics and in Yang-Mills theory in (2+1) dimensions

V. Ch. Zhukovsky[1], A. S. Razumovsky[1], and K. V. Zhukovsky[2]

Faculty of Physics, Moscow State University, 119899 Moscow, Russia

[1] Department of Theoretical Physics

[2] Department of Optics and Spectroscopy

E-mail: th180@phys.msu.su

Abstract

Vacuum effects in (2+1)-dimensional quantum electrodynamics (QED) with the topological Chern-Simons term are considered. The photon polarization operator is studied and the decay rate for the electron-positron photoproduction $\gamma \to e^+ e^-$ is presented as a function of the photon energy and external field strength. The radiatively induced electron mass shift in an external magnetic field is investigated both taking the topological Chern-Simons term into account and ignoring it. Moreover, the electron self-energy in topologically massive (2+1)-QED at finite temperature and density is studied. Finally, the parity breaking part of the action in the framework of the $SU(2) \times U(1)$ gauge field model at finite temperature is considered. The massive fermion contribution to the one-loop effective action in the background of the superposition of an abelian and a non-abelian gauge fields leading to parity breaking in the finite temperature (2+1)-quantum field theory is discussed.

Contents

1 Introduction

Investigations of quantum field theory models in low-dimensional spaces started with a number of discoveries made in late 1970's and in early 1980's. In 1979, studies of linear polymers were reported in [1], where it was stated that the main characteristics of the continuum model of polymer chains coincide with those of the already known one-dimensional models of quantum fields. Thus, the low-dimensional models proved to be very useful instruments for studying quasi-one-dimensional and quasi-two-dimensional media. Moreover, starting from the discovery of the integer quantum Hall effect in 1980 by von Klitzing and his collaborators [2], these models in the $(2+1)$-dimensional space-time have become especially popular. Recently, a close connection between certain predictions of the low-dimensional quantum field theory and a number of unusual phenomena detected experimentally in condensed matter physics has been revealed.

Odd-dimensional gauge theories have attracted much attention since 1981, when S. Deser, R. Jackiw and S. Templeton [3], [4], and N. Schonfield [5] demonstrated that, in the three dimensional space-time, a massive gauge invariant field theory can be constructed by adding (or obtaining due to fermion fluctuations) a topological Chern-Simons (CS) term S_{CS} to the action of matter and fields.

Topologically massive $(2+1)$-dimensional theories demonstrate a number of unusual properties. For instance, the finite mass of gauge fields leads to screening of both electric and magnetic fields [4]-[8], attraction of like charges becomes possible [7, 9], and the requirement of invariance with respect to topologically nontrivial gauge transformations in non-abelian theories leads to quantization of the parameter θ that plays the role of the gauge field mass [4].

Interesting properties of $(2+1)$-dimensional theories are related to statistics. There are examples of $(3+1)$-dimensional systems with anionic (fractional) statistics that is intermediate

between the Fermi and Bose statistics [10]. In the $(2+1)$-dimensional world, the anionic statistics, however, is realized in a unique way [11, 12].

As mentioned above, the $(2+1)$-dimensional models are not only formal illustrative examples, but they also have practical applications in the solid state physics, such as, for instance, high temperature superconductivity or the quantum Hall effect, which is explained in the framework of the two-dimensional anionic model [13]-[15]. The hypothesis that an anionic gas can obtain superconducting properties was proposed in [16, 17] after quasi-planar structures were discovered in high-temperature superconductors [18], and it was confirmed by calculations in [18, 19]. After that, the idea of anionic mechanism of high temperature superconductivity has become quite popular [20]-[23]. In this connection, the study of radiative effects in $(2+1)$–dimensional QED under various external conditions is on the agenda. In particular, the photon polarization operator and the Debye length in $(2+1)$–QED at finite temperature and vanishing chemical potential were studied in [24], and at zero temperature and finite chemical potential in [25].

Effects of external gauge fields [26]-[31], as well as of the gravitational field (see, e.g., [32]), together with the effects of finite temperature and nonzero chemical potential, in various (2+1)-dimensional quantum field models attracted much interest recently.

This, in particular, is related to the fact that many physical effects can take place only in the presence of an external field. For instance, the quantum Hall effect is explained basing on the peculiarities of the energy spectrum of a two-dimensional electron gas in a strong magnetic field, and an external magnetic field is responsible for superconductivity in a two-dimensional system. We also note that, as it was demonstrated in recent studies of radiative effects in (2+1)-dimensional theories [30]-[35], the CS topological term plays the role of an IR regulator parameter even in those cases when an external field is present, which in itself seems to be able to play the role of an IR regulator. In particular, the one-loop electron mass operator and the photon polarization operator in (2+1)-QED in an external magnetic field at finite temperature and density [30]-[34], as well as the quark mass operator in (2+1)-QCD in an external chromomagnetic field [35] were calculated. It was demonstrated that these quantities are described by nonanalytic functions of the external field. With consideration for the CS term, the exact solutions of the non-abelian YM field equations in (2+1) dimensions were found and one-loop vacuum corrections to the effective lagrangian of these fields due to fluctuations of gauge and fermion fields were calculated [36]. We also note that the problem of vacuum effects in low dimensional field theories is of special interest due to the development of non-abelian gauge field theories at high temperatures and in strong external fields, where the effective dimensional reduction takes place (see, e.g., [37] and references therein). Recently, various effects in odd-dimensional theories have been studied with consideration for the influence of external conditions (see, e.g., investigations of the photon polarization operator and the electron mass operator in the $(2+1)$-dimensional quantum electrodynamics in a magnetic field at finite temperature and density [38, 31, 39]).

It should also be mentioned that the problem of dynamical generation of the Chern-Simons term in certain models was considered in detail in [40] and [41]. Note also the discussion of the role of the finite matter density in the effect of dynamical generation of the CS term in odd-dimensional gauge theories (see, e.g.,[42, 43]), which demonstrates that the scope of the study of topological problems in gauge theories is still far from being exhausted.

The fundamental property of the gauge field action in the non-abelian case is its noninvari-

ance with respect to the so called "large" (homotopically nontrivial) gauge transformations. Therefore, to make the path integral, which involves $\exp(iS_{CS})$, invariant under these transformations, the requirement that the CS coefficient (the topological mass) is quantized in units of $g^2/(4\pi)$ should be adopted. However, the above holds only for the zero temperature quantum fields. When the temperature is finite, situation becomes quite different. This relates to the fact that perturbative corrections to the CS term are non-quantized continuous functions of temperature [44]. As a result, "large" gauge invariance would be lost at finite temperature. Nevertheless, as it was demonstrated by S. Deser, L. Griguolo, and D. Seminara [45]–[48], even though the CS term itself may violate "large" gauge invariance, there exist other terms in the effective action, which can compensate for this violation and make the total effective action gauge invariant under both small and large gauge transformations. At first, it was explicitly shown for the (0+1) dimensional model [49, 50]. Then, this analysis was generalized to (2+1) dimensional fermions interacting with Abelian [45, 46, 47, 51] and non-Abelian [45, 48] gauge backgrounds. There, in the framework of the non-perturbative approach, it was clearly demonstrated, that even though the perturbative expansion leads to a non-quantized temperature-dependent CS coefficient, the full effective action can be made invariant under "large" gauge transformations (LGT) at any temperature, once the suitable regularization of the Dirac operator determinant is applied. As a result, there arises a parity anomaly as a price for the restoration of the invariance.

Of special interest are recently proposed various modifications of the (3+1)-dimensional QED. For instance, a Chern-Simons like term can be added to the QED Lagrangian [52] with the coupling of the dual electromagnetic field tensor to a certain fixed 4-vector, or an additional Lorentz-noninvariant term can be introduced in the fermionic Lagrangian [53, 54].

In the present review, as an illustration of the above observations, we present some results of calculations of vacuum effects in (2+1)-dimensional QED. In Section 2, some basic results of the (2+1)-dimensional QED are formulated. Further, the study of the photon polarization operator is presented and the rate of the photo-production of electron-positron pair $\gamma \to e^+ \, e^-$ as a function of the photon energy and of the external field strength is calculated (Section 3). In Section 4, we investigate the radiatively induced electron mass shift in an external magnetic field with the topological Chern-Simons term as well as without it. The electron vacuum energy in the topologically massive (2+1)-QED at finite temperature and density is studied in Section 4.

Section 5 is devoted to the topological effects in (2+1)-dimensional $SU(2) \times U(1)$-gauge field theory with the superposition of the Yang-Mills field ($SU(2)$ group) and the Maxwell field ($U(1)$ group). As an example, we have calculated the exact expression for the parity breaking part of the effective action in this model. In conclusion, the analysis of the results obtained is presented.

2 (2+1)-Dimensional Quantum Electrodynamics

The topologically massive $(2+1)$-dimensional quantum electrodynamics is described by the following Lagrangian [4]:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left[\gamma^\mu (i\partial_\mu - eA_\mu) - m \right] \psi + \frac{1}{4} \theta \varepsilon^{\mu\nu\alpha} F_{\mu\nu} A_\alpha, \tag{1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, and matrices γ^μ ($\mu = 0, 1, 2$), coincidig with the Pauli matrices, satisfy the folloing relations

$$\gamma^0 = \sigma^3, \quad \gamma^1 = i\sigma^1, \quad \gamma^2 = i\sigma^2, \tag{2}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad \gamma^\mu\gamma^\nu = g^{\mu\nu} - i\varepsilon^{\mu\nu\lambda}\gamma_\lambda, \quad g^{\mu\nu} = \mathrm{diag}\,(1, -1, -1)\,. \tag{3}$$

The last term in (1) is the so called topological Chern-Simons term with the coefficient θ as the gauge field mass.

Under the gauge transformations $A_\mu \to A_\mu + \dfrac{1}{e}\partial_\mu\alpha$, $\varphi \to \mathrm{e}^{i\alpha}\,\varphi$, $\bar{\varphi} \to \mathrm{e}^{-i\alpha}\,\bar{\varphi}$, the Lagrangian is changed by adding the full derivative $\mathcal{L} \to \mathcal{L} + \partial_\rho\Big(\dfrac{\theta}{4e}\varepsilon^{\rho\mu\nu}F_{\mu\nu}\alpha\Big)$. The Chern-Simons term together with the fermion mass term lead to breaking P and T symmetries, while PT and CPT symmetries are conserved.

It should be mentioned that even if no CS term is present initially in the Lagrangian (1) of the theory with broken P and T symmetries, this term is generated by quantum fluctuations of massive fermions. The magnitude of this terms significantly depends both on finite temperature and density [55, 56], and on the external field strength [30]-[34]. In the one-loop approximation, this can be made clear by calculating the photon polarization operator (PO). First of all, consider the photon propagator in QED with the topological term. It can be written in the following form [4] (in the general Landau gauge):

$$D_{\mu\nu}\,(k) = -\frac{i}{k^2 - \theta^2 + i\varepsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + i\theta\frac{\varepsilon_{\mu\nu\lambda}k^\lambda}{k^2}\right). \tag{4}$$

This expression clearly demonstrates that the mass of the gauge field is equal to θ.

The one-loop photon PO in (2+1)-QED is determined by the expression:

$$\Pi_{\mu\nu}(k) = e^2\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}\Big[\gamma_\mu S_c\Big(x'x\Big)\gamma_\nu S_c\Big(x\,x'\Big)\Big]. \tag{5}$$

It should be emphasized that the polarization operator $\Pi_{\mu\nu}(k)$ in (3+1)-QED is a symmetric tensor, while in the (2+1)-QED it is represented as a sum of two terms

$$\Pi_{\mu\nu} = \Pi^S_{\mu\nu} + \Pi^A_{\mu\nu}, \tag{6}$$

one of them $\Pi^S_{\mu\nu}$ being symmetric, and the other $\Pi^A_{\mu\nu}$ antisymmetric. The so called dynamically induced (antisymmetric) part of the CS term in the PO has the following structure:

$$\Pi^A_{\mu\nu}(q) = i\varepsilon_{\mu\nu\alpha}q^\alpha\Pi^A(q^2), \tag{7}$$

Here, the quantity $\Pi^A(q^2)$ at $q^2 = 0$ determines the Chern-Simons mass induced by radiative effects [4, 56]

$$\theta_{ind} = \lim_{q\to 0}\Pi^A(q^2). \tag{8}$$

It should be mentioned that higher order terms of the perturbation theory do not contribute to θ_{ind} (see also [57, 58]).

Calculations of the photon PO as well as of the electron mass operator require the knowledge of the fermion propagator in the external magnetic field, which is determined by the following equation:

$$\left[\left(i\partial_\mu - eA_\mu\right)\gamma^\mu - m\right] G\left(x, x'\right) = \delta^3\left(x - x'\right), \tag{9}$$

where $G(x, x')$ and $S(x, x')$ are connected by the relation

$$G\left(x, x'\right) = \left[\left(i\partial_\mu - eA_\mu\right)\gamma^\mu + m\right] S\left(x, x'\right). \tag{10}$$

For a constant magnetic field, given by the potential

$$A^\mu = \left(0, 0, x^1 H\right), \; F_{12} = -F_{21} = H, \tag{11}$$

calculations of the electron propagator are conveniently performed by the Fock-Schwinger proper-time method [59, 60], resulting in the following expression [61]:

$$\begin{aligned} G\left(x, x'\right) = {} & \frac{\mathrm{e}^{i\pi/4}\, eH}{(4\pi)^{3/2}} \Big[\gamma^\mu\left(i\,\partial_{\mu_x} - eA_\mu\left(x\right)\right) + m\Big] \int\limits_0^\infty \frac{ds}{s^{1/2}} \exp\Big(- ie \int\limits_x^{x'} d\xi^\mu A_\mu\left(\xi\right)\Big) \times \\ & \times \Big(I\, \cot(eHs) - i\sigma^3\Big) \exp\Big(-\frac{i}{4}\Big[\frac{\Delta x_0^2}{s} - \Delta x^2 eH \cot(eHs)\Big] - is\left(m^2 - i\varepsilon\right)\Big), \end{aligned} \tag{12}$$

where m is the electron mass, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\Delta x_1 = x_1 - x_1'$, $\Delta x_2 = x_2 - x_2'$, $\Delta x_0 = x_0 - x_0'$, $\Delta x^2 = \left(\Delta x_1\right)^2 - \left(\Delta x_2\right)^2$. The substantial difference between the above expression and the expression for the electron propagator in (3+1)-QED is in the preexponential factor in the integral over s: it is $1/s^{1/2}$ instead of $1/s$ in the (3+1)-QED.

The important feature of the fermion spectrum in the external field in the (2+1)-dimensional QED is its discreteness:

$$p_0 = \pm\left(m^2 + 2\left|eH\right| k\right)^{1/2}, \; k = 0, 1, 2, \ldots. \tag{13}$$

This spectrum is typical for fermions in space-times of reduced dimensions, since the degree of freedom that corresponds to motion along the field is missing in this case, contrary to the (3+1)-dimensional QED. In that last case, the fermion spectrum is given by the expression that depends also on the projection p_3 of the electron momentum on the direction of the field vector, taking continuous values, i.e.,

$$p_0 = \pm\left(m^2 + 2\left|eH\right| n + p_3\right)^{1/2}, \; n = 0, 1, 2, \ldots.$$

One more special feature of the (2+1)-QED should be mentioned: contrary to (3+1)-QED with the dual field tensor

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho} F^{\lambda\rho} \tag{14}$$

no such tensor can be constructed in the (2+1)-QED, due to the absence of the 4^{th} rank tensor like $\varepsilon_{\mu\nu\lambda\rho}$.

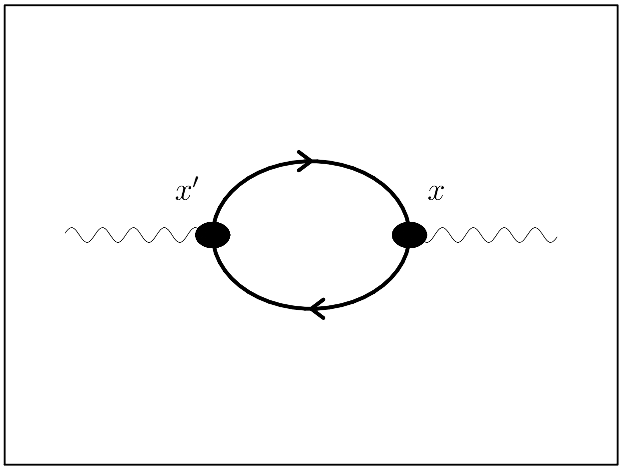

Figure 1: Photon polarization operator in the one-loop approximation.

3 Photon polarization operator in (2+1)-dimensional QED

3.1 Polarization operator and photon elastic scattering amplitude in a constant magnetic field

In order to calculate the photon elastic scattering amplitude, consider the photon polarization operator, written in the one-loop approximation according to (12). The equivalent graphical representation is given in Fig.1 In the constant magnetic field, determined by the potential (11), electron propagator (12) can be transformed into the following form [30]:

$$S\left(x,x'\right) = -\frac{\mathrm{e}^{-i\pi/4}\,h}{\left(4\pi\right)^{3/2}}\int\limits_0^\infty \frac{ds}{\sqrt{s}}\frac{\mathrm{e}^{-ism^2}}{\sin\left(hs\right)}\exp\left(-i\frac{X_0^2}{4s}+i\frac{X_\perp^2 h\,\cot\left(hs\right)}{4}-iuY\frac{h}{2}\right)\times$$
$$\times\left[\frac{1}{2s}\left(\gamma^0 T-\frac{hs}{\sin\left(hs\right)}\left(\gamma X\right)_\perp \mathrm{e}^{ihs\gamma^0}\right)+m\right]\mathrm{e}^{-ihs\gamma^0}, \tag{15}$$

where $h = eH$ and

$$X^\mu = x^\mu - x'^\mu,\quad u = x^1 + x'^1,\quad Y = x^2 - x'^2,\quad X_\perp^2 = \left(x^1 - x'^1\right)^2 + \left(x^2 - x'^2\right)^2. \tag{16}$$

Calculation of the photon polarization operator (5) is performed similar to the (3+1)-dimensional QED [62]. In the (2+1)-dimensional QED, (5) depends on the kinetic momentum operator $\hat{P}_\mu = i\partial_\mu - ieA_\mu$ and operators $F^{\mu\nu}\hat{P}_\nu$, $F^{\ \mu\nu}F_{\nu\lambda}\hat{P}^\lambda$, and thus, it commutes with operator $\hat{P}_\mu$ in a constant magnetic field. Therefore, the photon PO turns out to be diagonal in the momentum k-space:

$$\Pi^{\mu\nu}\left(k,k'\right) = \int \Pi_{\mu\nu}\left(x,x'\right)\exp\Big(-i\left(kx-kx'\right)\Big)dxdx' = \left(2\pi\right)^3\delta\left(k-k'\right)P_{\mu\nu}\left(k\right) \tag{17}$$

As was already mentioned above, in contrast to (3+1)-QED, where the polarization operator is symmetrical, it is represented in (2+1)-QED as a sum of symmetric and antisymmetric terms (6).

Consider first the symmetric part $P^s_{\mu\nu}\left(k,H\right)$ and expand it over its eigenvectors $V^{(i)}_\mu$:

$$P^s_{\mu\nu}\left(k,H\right)=\sum_{i=1}^{3}\lambda_i\frac{V^{(i)}_\mu V^{(i)}_\nu}{\left|V^i\right|^2}\quad , \tag{18}$$

where

$$V^{(1)}_\mu\equiv l_\mu=\frac{F_{\mu\nu}k^\nu}{\left(F^2_{\lambda\rho}/2\right)^{1/2}},\quad V^{(2)}_\mu\equiv v_\mu=k_\mu+\frac{k^2F_{\mu\nu}l^\nu}{l^2\left(F^2_{\lambda\rho}/2\right)^{1/2}},\quad V^{(3)}_\mu=k_\mu, \tag{19}$$

$$\frac{l_\mu l_\nu}{l^2}+\frac{v_\mu v_\nu}{v^2}+\frac{k_\mu k_\nu}{k^2}=g_{\mu\nu}.$$

One of the eigenvalues of $P^s_{\mu\nu}\left(k,H\right)$ vanishes, $\lambda_3=0$, since the polarization operator $\Pi_{\mu\nu}$ is transversal due to its gauge invariance:

$$k_\mu\Pi^{\mu\nu}=0 \tag{20}$$

Since the dual tensor can not be constructed in (2+1) dimensions, the eigenvector

$$V^{(4)}_\mu=\frac{\tilde{F}_{\mu\nu}k^\nu}{\left(F^2_{\lambda\rho}/2\right)^{1/2}} \tag{21}$$

does not exist either, so the number of eigenvectors $V^{(i)}_\mu$ of $P^s_{\mu\nu}\left(k,H\right)$ is also reduced to three, and only two of the eigenvalues λ_i are not vanishing. The symmetric part of the polarization operator can be expanded as follows:

$$P^s_{\mu\nu}\left(k,H\right)=\lambda_1\frac{l_\mu l_\nu}{l^2}+\lambda_2\frac{v_\mu v_\nu}{v^2}=\left(\lambda_1-\lambda_2\right)\frac{l_\mu l_\nu}{l^2}+\lambda_2\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right) \tag{22}$$

where

$$l_\mu=\left(0,k_2,-k_1\right),\quad v_\mu=\left(k_0,\frac{k_0^2}{\mathbf{k}^2}k_1,\frac{k_0^2}{\mathbf{k}^2}k_2\right). \tag{23}$$

Calculating the eigenvalues λ_1, λ_2 from the eigenvalue equation in the form

$$\lambda_1=\frac{1}{l^2}P^s_{\mu\nu}l^\mu l^\nu,\quad \lambda_2=\frac{1}{v^2}P^s_{\mu\nu}v^\mu v^\nu, \tag{24}$$

we obtain for them the following result that takes account of the external field exactly and allows to write the symmetric part of the photon polarization operator (22), (23):

$$\begin{aligned}\left(\begin{array}{c}\lambda_1\\ \lambda_2\end{array}\right)&=-\frac{\mathrm{e}^{i\pi/4}\,e^2h}{\left(4\pi\right)^{3/2}}\int_0^\infty ds\int_0^\infty dt\frac{1}{\left(s+t\right)^{1/2}\sin\left(s+t\right)}\times\\ &\times\exp\left(i\Big[\frac{st}{s+t}k_0^2-\frac{\sin\left(hs\right)\sin\left(ht\right)}{h\sin\left(h(s+t)\right)}\mathbf{k}^2-m^2\left(s+t\right)\Big]\right)\left(\begin{array}{c}A_1\\ A_2\end{array}\right),\end{aligned} \tag{25}$$

$$A_1 = 2m^2\cos\left(h\left(t-s\right)\right) - 2imk_0\sin\left(h\left(s-t\right)\right) + \left(-i + \frac{2stk_0^2}{s+t}\right)\frac{\cos\left(h\left(s-t\right)\right)}{s+t} -$$

$$-2\frac{\sin\left(hs\right)\sin\left(ht\right)}{\sin^2\left(h\left(s+t\right)\right)}\mathbf{k}^2, \tag{26}$$

$$A_2 = A_1 + 2i\frac{\mathbf{k}^2}{k^2}\left[\frac{h}{\sin\left(h\left(s+t\right)\right)} - \frac{\cos\left(hs\right)\cos\left(ht\right)}{s+t}\right] + 4\frac{\mathbf{k}^2}{k^2}m^2\sin\left(hs\right)\sin\left(ht\right) -$$

$$-4k_0^2\frac{\mathbf{k}^2}{k^2}\left[\frac{t\sin\left(hs\right)+s\sin\left(ht\right)}{2\left(s+t\right)\sin\left(h\left(s+t\right)\right)} - \frac{\sin\left(hs\right)\sin\left(ht\right)}{\sin^2\left(h\left(s+t\right)\right)} - \frac{st}{\left(s+t\right)^2}\cos\left(hs\right)\cos\left(ht\right)\right]. \tag{27}$$

The antisymmetric part of the PO, calculated with account for formulas (2), (5), (7), (15)-(17), becomes

$$P_a^{\mu\nu}\left(k,H\right) = 2im\,\mathrm{e}^{i\pi/4}\frac{e^2H}{\left(4\pi\right)^{3/2}}\varepsilon^{\mu\nu\lambda}k_\lambda\int\limits_0^\infty ds\int\limits_0^\infty dt\frac{\cos\left(h\left(s-t\right)\right)}{\sqrt{s+t}\sin\left(h\left(s+t\right)\right)}\times$$

$$\times\exp\left(i\Big[\frac{st}{s+t}k_0^2 - \mathbf{k}^2\frac{\sin\left(hs\right)\sin\left(ht\right)}{h\sin\left(h\left(s+t\right)\right)} - m^2\left(s+t\right)\Big]\right) \tag{28}$$

This part of the PO determines the induced Chern-Simons mass in (2+1)-QED, according to (7), (8). Thus, the Chern-Simons term can be induced not only by the finite temperature and density ($T \neq 0, \mu \neq 0$) [55, 56], but also by the effects of the external field. This is the important conclusion in considering the topologically massive QED, especially in the case, when the initial Lagrangian does not contain a term with the Chern-Simons mass.

The polarization operator, calculated on the mass shell, $k^2 = 0$, determines the photon elastic scattering amplitude:

$$T = \frac{1}{2\omega}e_\mu P_{\mathrm{reg}}^{\mu\nu}e_\nu, \tag{29}$$

where $\omega = k + 0 = |\mathbf{k}|$ is the photon energy, e_μ is the photon polarization vector. The photon PO is renormalized in the standard way:

$$P_{\mathrm{reg}}^{\mu\nu}\left(k,H\right) = P_{\mu\nu}\left(k,H\right) - P_{\mu\nu}\left(k,H=0\right) + P_{\mu\nu}\left(k\right), \tag{30}$$

where $P_{\mu\nu}(k)$ is the renormalized polarization operator in the absence of the field. On the mass shell, $m = 0$, the eigenvector $v_\mu^{(2)} = k_\mu$ and hence the PO in (2+1)-QED is determined by a single vector of linear polarization. It can be represented in the following form:

$$e_\mu = \frac{l_\mu}{\left(-l^2\right)^{1/2}} = \frac{1}{|\mathbf{k}|}\left(0, k_2, -k_1\right), \quad l_\mu = \frac{F_{\mu\nu}k^\nu}{\left(F_{\lambda\rho}^2\right)^{1/2}}. \tag{31}$$

Finally, we obtain the photon elastic scattering amplitude:

$$T = \mathrm{e}^{i\pi/4}\frac{e^2m}{\left(4\pi\right)^{3/2}\omega}\sqrt{2\frac{H_0}{H}}\int\limits_{-1}^{1}dv\int\limits_0^\infty\sqrt{\rho}d\rho\exp\left(-2i\rho\frac{H_0}{H}\right)\times$$

$$\times\left[\frac{A\left(\rho,v\right)}{\sin\left(2\rho\right)}e^{i\phi} - \frac{1}{2\rho}\left(1 - \frac{i}{4\rho}\frac{H}{H_0}\right)\right] \tag{32}$$

where $H_0 = m^2/e$ is the (2+1)-analogue of the Schwinger critical magnetic field introduced initially in the (3+1)-dimensional electrodynamics, and the functions ϕ and $A(\rho,v)$ are given by the following expressions:

$$\phi = \frac{\omega^2}{h}\left[\frac{\rho\left(1-v^2\right)}{2} - \frac{\sin\left(\rho\left(1-v\right)\right)\sin\left(\rho\left(1+v\right)\right)}{\sin 2\rho}\right], \tag{33}$$

$$A\left(\rho,v\right) = \cos\left(2\rho v\right) - i\frac{\omega}{m}\sin\left(2\rho v\right) + \frac{H}{4H_0\rho}\cos\left(2\rho v\right)\left(-i + \frac{\omega^2\rho\left(1-v^2\right)}{eH}\right) - \\ - \left(\frac{\omega}{m}\right)^2 \frac{\sin\left(\rho\left(1+v\right)\right)\sin\left(\rho\left(1-v\right)\right)}{\sin^2\left(2\rho\right)}. \tag{34}$$

It is worth mentioning that the above expression for the photon elastic scattering amplitude takes both the photon energy ω and the external field intensity H exactly into account, and hence, it is valid for the fields of arbitrary intensity.

3.2 Electron-positron pair photo-production in an external magnetic field

The imaginary and real parts of the elastic scattering amplitude in a constant magnetic field (29) yield, via the optical theorem, the rate of electron-positron pair photo-production and the photon mass squared, respectively:

$$w = -2\mathrm{Im}(T), \tag{35}$$

$$\delta\left(m^2\right) = 2\omega\mathrm{Re}(T) \tag{36}$$

Consider now the case of relatively weak magnetic fields and high energies of photons:

$$H \ll H_0, \quad m \ll \omega. \tag{37}$$

Expanding trigonometric functions in (32)-(34) in the domain $\rho \ll 1$, which provides the main contribution to the amplitude in (32), we can rewrite the photon scattering amplitude as follows:

$$T = -i\, \mathrm{e}^{i\pi/4}\, \frac{e^2 m}{\left(4\pi\right)^{3/2}\omega} \int\limits_1^\infty \frac{du}{u^{3/2}\sqrt{u-1}} \left[1 + \frac{8u-5}{3}\right] \left(\frac{\chi}{4u}\right)^{1/3} G'\left(\zeta\right), \tag{38}$$

where parameter χ is determined by the relation

$$\chi = \frac{H\omega}{H_0 m} = \sqrt{-\frac{e^2\left(F_{\mu\nu}k^\nu\right)^2}{m^6}}. \tag{39}$$

We have also introduced the function

$$G\left(\zeta\right) = \int\limits_0^\infty \sqrt{y}dy \exp\left(-iy\zeta - iy^3/3\right), \tag{40}$$

where $\zeta = (4u/\chi)^{2/3}$. Thus, in the above range of comparatively weak magnetic fields and high photon energies, the photon scattering amplitude depends on the external field strength and photon energy via only single parameter χ. The function $G(\zeta)$ is a generalization of the well known Airy function $f(\zeta) = i\int\limits_0^\infty dx \exp(-iyx - ix^3/3)$ in the amplitude of elastic photon scattering in the (3+1)-QED in the case $H \ll H_0$, $m \ll k_\perp$. Now, with the help of Mellin transformations with the parameter $\lambda = 4\sqrt{3}/\chi$, we obtain the elastic scattering amplitude:

$$T(\lambda) = \frac{1}{2\pi i}\int\limits_{\gamma-i\infty}^{\gamma+i\infty} ds \lambda^{-s} T(s), \tag{41}$$

$$\begin{aligned} T(s) = &-\frac{e^2 m \Gamma(1/2)}{(4\pi)^{3/2}\omega}\frac{36}{5}\exp\left(-\frac{i\pi s}{2}\right)\Gamma\left(1-\frac{s}{2}\right)\Gamma\left(\frac{3s}{2}-\frac{1}{2}\right)\times \\ &\times\left[\frac{4\Gamma(s)}{\Gamma(s+1/2)} - \frac{\Gamma(s+1/2)}{\Gamma(s+3/2)}\right] \end{aligned} \tag{42}$$

The integral (41), according to [30], can be expressed in terms of Meijer G-functions [63].

In the framework of the general study of the structure of the photon polarization operator in (2+1)-dimensional QED in a constant magnetic field, expressions for the symmetric and antisymmetric parts of the PO were obtained in [28, 30].

In the case of relatively weak fields and high photon energies, when

$$H \ll H_0 = \frac{m^2}{e},\ \omega \gg m, \tag{43}$$

the photon elastic scattering amplitude in (2+1)-QED has the following asymptotics [30]

$$T = \begin{cases} -i\dfrac{e^2 m \exp\left(-i\frac{\pi}{6}\right)}{\omega(4\pi)^{3/2}}\dfrac{24}{5}\Gamma\left(\dfrac{1}{2}\right)\Gamma\left(\dfrac{1}{3}\right)\left(\dfrac{\chi}{4\sqrt{3}}\right)^{1/3}, & \chi \gg 1, \\ -\dfrac{e^2 m}{360\pi\omega}\chi^2 - i\dfrac{e^2 m}{8\pi\omega}\left(\dfrac{3\chi\pi}{8}\right)^{1/2}\exp\left(-\dfrac{8}{3\chi}\right), & \chi \ll 1, \end{cases} \tag{44}$$

Compare the results of (44) with the analogous results of (3+1)-QED [62]. For $\chi \gg 1$, the photon elastic scattering amplitude (both its real and imaginary parts) in (2+1)-QED is proportional to $\chi^{1/3}$, while in (3+1)-QED it grows as $\propto \chi^{2/3}$. Thus, for $\chi \gg 1$, the lowering of space-time dimensionality effectively leads to suppression of the growth of the one-loop contribution to the scattering amplitude by the factor $\chi^{1/3}$. Nevertheless, no regular correspondence between changing the space-time dimensionality and the dependence of the amplitude A on parameter χ can be detected. In fact, for $\chi \ll 1$, the imaginary part of A that according to the optical theorem determines the rate of electron-positron pair production, in (2+1)-QED is proportional to $\chi^{1/2}\exp\left(-\dfrac{8}{3\chi}\right)$, while in (3+1)-QED the preexponential factor is $\propto \chi$. As for the real part of the amplitude T, it appears to be proportional to χ^2 for $\chi \gg 1$ in both cases. The above conclusions demonstrate that the magnetic properties of photons change significantly with the reduction of the space-time dimensionality from (3+1) to (2+1).

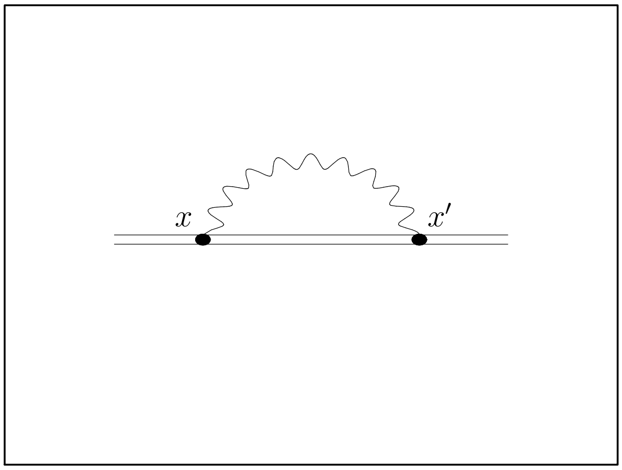

Figure 2: The electron ground state energy shift ε_0 in an external magnetic field in the one-loop approximation.

4 Radiative electron energy shift in (2+1)-QED

4.1 Electron mass shift in (2+1)-QED in external magnetic field without Chern-Simons term

The radiative shift of the electron ground state energy in an external constant magnetic field H in the framework of (2+1)-QED was studied in [31], where the cases with $\theta = 0$ and $\theta \neq 0$ (topologically massive QED) were studied separately.

The radiative shift $\Delta\varepsilon_0$ of the electron ground state energy ε_0 in an external magnetic field in the one loop approximation is depicted in Fig. 2. It is described by the following integral:

$$\Delta\varepsilon = \frac{1}{T}\int d^3x d^3x' \bar{\varphi}_n\left(x\right) M\left(x, x'\right) \varphi_n\left(x'\right) \tag{45}$$

where

$$M\left(x, x'\right) = -ie\gamma^{\mu} G\left(x, x'\right) \gamma^{\nu} D_{\mu\nu}\left(x, x'\right) \tag{46}$$

is the mass operator in the one-loop approximation, T is a sufficiently long time interval, during which the process proceeds, φ_n is the electron wave function in the external electromagnetic field, $G(x, x')$ and $D_{\mu\nu}(x, x')$ are the electron and photon propagators in the external field.

Consider the electron ground state, when $\varepsilon_0 = m$, where m is the electron mass, the electron charge $e = -e_0 < 0$.

We will consider for the external field using the exact solutions of the Dirac equation as the electron wave function and also the exact propagators of the particles in external field. For the field given by the potential in the gauge (11), we employ the electron propagator in the form (12). The two-component wave function of an electron in the magnetic field is as follows:

$$\varphi_0\left(x\right) = L^{-1/2}\left(h/\pi\right)^{1/4} \exp\left[-imt - h\left(x_1\right)^2/2\right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{47}$$

where L is the normalization length. First, we consider the case without a topological term in the QED Lagrangian, which is most close to QED in (3+1) dimensions. Recall that in the (2+1)-QED the electron spectrum in a magnetic field is purely discreet (13). We write the photon propagator in the Feynman gauge:

$$D_{\mu\nu}(k) = \frac{g_{\mu\nu}}{k^2 + i0}, \tag{48}$$

where k is the virtual photon momentum. Calculations, similar to those for the (3+1)-QED case, and subtraction of the divergent term $\Delta\varepsilon\,|_{H\to 0}$ yield the following integral representation of the electron energy shift:

$$\Delta\varepsilon_0 = \frac{e^2}{8\pi^{3/2}\sqrt{f}} \int\limits_0^\infty \frac{dy}{\sqrt{y}} \int\limits_0^1 \frac{du}{\sqrt{u}} \exp\left(-\frac{uy}{f}\right) \left[\frac{2 - u + 2u\,\mathrm{e}^{-2y}}{1 - u + \dfrac{u \sinh(y)}{y}\,\mathrm{e}^{-y}} - (2 + y) \right], \tag{49}$$

where the field parameter is $f = e_0 H/m^2$. Now consider the asymptotic behavior of $\Delta\varepsilon_0$ in the case of strong, $f \gg 1$, and weak, $f \ll 1$, fields.

In the case $\theta = 0$, the asymptotic expressions for the radiative electron mass shift have the form [31]:

$$\triangle\,\varepsilon_0 = \begin{cases} \dfrac{e^2}{8\pi} f \left(2 + \ln\left(\dfrac{f}{2}\right)\right), \; f = \dfrac{eH}{m^2} \ll 1, \\ \dfrac{e^2}{8\pi} \ln(2f), \;\; f \gg 1. \end{cases} \tag{50}$$

It is convenient to introduce the so called magnetic susceptibility, defined as follows:

$$\chi_{2+1} = \frac{\partial}{\partial f} \Delta\varepsilon_0. \tag{51}$$

Then, we obtain it in two limiting cases:

$$\chi_{2+1} = \frac{\partial}{\partial f}\Delta\varepsilon_0 = \frac{e^2}{8\pi} \begin{cases} \ln(f), \;\; f \ll 1, \\ \dfrac{1}{f}, \;\; f \gg 1. \end{cases} \tag{52}$$

Comparing the asymptotic behavior of the magnetic susceptibility $\chi = \dfrac{\partial\varepsilon_0}{\partial f}$ with the corresponding results for (3+1)-QED [62]:

$$\chi_{3+1} = \frac{e^2 m}{(4\pi)^2} \begin{cases} -1, \;\; f \ll 1, \\ \left(\dfrac{2}{f}\right) \ln(2f), \;\; f \gg 1, \end{cases} \tag{53}$$

we can observe the following:
1) in (2+1)-QED in strong field ($f \gg 1$), it decreases faster, i.e., ($\propto f^{-1}$), than in (3+1)-QED $\left(\propto \dfrac{\ln(f)}{f}\right)$;
2) in the weak field limit in (3+1)-QED the result for χ is finite and does not depend on the

field in the limit $f \ll 1$, while in (2+1)-QED, the effective magnetic susceptibility diverges at the origin as $\ln(f)$. This divergence, according to [31], is cancelled by consideration for the Chern–Simons term in (1).

Thus, lowering the space-time dimensionality leads to considerable changes in the behavior of the magnetic properties of electrons and to changes in radiative effects that accompany the photon propagation in a constant external field.

4.2 Electron mass shift in topologically massive (2+1)-QED in external magnetic field

Now let us turn to the topologically massive QED with the Chern-Simons term $\theta \neq 0$ [31], making use of the photon propagator in the Landau gauge (4). Then, instead of the topologically trivial case with $\theta = 0$ (49), we obtain the following expression for the electron mass shift in a magnetic field in topologically massive (2+1)-QED:

$$\Delta\varepsilon_0 = \frac{e^2}{8\pi^{3/2}\sqrt{f}}\int\limits_0^1 \frac{du}{\sqrt{u}}\int\limits_0^\infty \frac{dt}{\sqrt{t}}\,\mathrm{e}^{-ut/f}\left\{\frac{1}{F}\left[\mathrm{e}^{-v}\left(2-u+2u\,\mathrm{e}^{-2t}\right)+\right.\right.$$
$$\left.\left.+\frac{\mathrm{e}^{-v}-1}{v}\bar{u}\left(1-u\Big(\frac{3}{2}+\frac{\mathrm{e}^{-2t}}{F}+\frac{t\,(2-u)}{f}\Big)+\frac{\theta}{m}\,\mathrm{e}^{-2t}\left(1-\frac{2ut}{f}+\frac{2}{F}\right)\right)\right]-R_0\right\}, \tag{54}$$

where

$$R_0 = \mathrm{e}^{-v}\,(2+u)+\frac{\mathrm{e}^{-v}-1}{v}\bar{u}\left[1-u\left(\frac{5}{2}+\frac{t\,(2-u)}{f}\right)+\frac{\theta}{m}\left(3-\frac{2ut}{f}\right)\right], \tag{55}$$

$$\bar{u} = 1-u,\ v = \frac{\theta^2}{H}\frac{\bar{u}t}{u},\ F = \bar{u}+u\frac{\operatorname{sh} t}{t}\,\mathrm{e}^{-t}. \tag{56}$$

The above expression for the $\Delta\varepsilon_0$ accounts for the external field effect exactly. Note that the $\Delta\varepsilon_0$ is the function of two parameters: the field parameter $f = e_0H/m^2$ and the mass parameter

$$\mu = \frac{\theta}{m}. \tag{57}$$

Considering the asymptotic behavior of the electron energy shift in a weak magnetic field: $f \ll 1$, $\mu \ll 1$, we obtain the result:

$$\Delta\varepsilon_0 = \frac{e^2}{8\pi}f\left(2+\ln\left(\frac{f}{2}\right)+f\,(\lambda)\right), \tag{58}$$

where

$$\lambda = \frac{\mu}{f},\ f\,(\lambda) = (1+\lambda)^2\ln(1+\lambda)-\lambda-\lambda\,(\lambda+2)\ln(\lambda) \tag{59}$$

and λ has an arbitrary value. When $\lambda = 0 (\theta = 0)$, $f(\lambda)$ vanishes and (58) reduces to (4), calculated without Chern-Simons term in the Lagrangian, i.e. with $\theta = 0$. It is in accordance with the gauge invariance of the theory (compare (48) and (4)).

When $\lambda \gg 1$, the energy shift (58), (59) writes as follows:

$$\Delta\varepsilon_0 = \frac{e^2}{8\pi}f\Big(\ln(\mu)+\mathrm{const}\Big), \quad f \ll \mu \ll 1. \tag{60}$$

Hence, the magnetic susceptibility χ_{2+1} in weak magnetic fields, when $f \to 0$, becomes finite due to the topological term $\theta \neq 0$ that cures logarithmic divergence of the magnetic susceptibility χ_{2+1}, taking place for $\theta = 0$.

4.3 Electron self-energy in (2+1)-dimensional topologically massive QED at finite temperature and density

Here, we will calculate the contributions of the finite temperature and density to the electron self energy in (2+1)-QED with the Chern-Simons term, represented in the Lagrangian (1). The dependence of the dispersion law on the finite temperature and matter density

$$E^2 = \mathbf{p}^2 + m^2 + \Delta\left(E^2\right) \tag{61}$$

is represented by the same formulas [34] as in (3+1)-QED:

$$\Delta\left(E^2\right) = -\frac{1}{2}\mathrm{Tr}\Big[\left(\hat{p} + m\right)\Sigma\left(p\right)\Big]. \tag{62}$$

We use the following expression for the electron mass operator

$$\Sigma\left(p\right) = ie^2\int\frac{d^3k}{\left(2\pi\right)^3}\gamma^\mu S\left(p-k\right)\gamma^\nu D_{\mu\nu}\left(k\right) \tag{63}$$

and perform our calculations in the framework of the the real-time finite temperature technique. We choose the temporal electron and photon Green functions in the Landau gauge [34]

$$D_{\mu\nu}\left(k\right) = -\left[\frac{1}{k^2-\theta^2+i0} + 2\pi\frac{\delta\left(k^2-\theta^2\right)}{\mathrm{e}^{(|k_0|/T)}-1}\right]\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2+i0} + i\theta\varepsilon_{\mu\nu\lambda}\frac{k^\lambda}{k^2+i0}\right), \tag{64}$$

$$S\left(p\right) = i\frac{\hat{p}+m}{p^2-m^2+i\varepsilon} - 2\pi\delta\left(p^2-m\right)\left(\hat{p}+m\right)\left[\frac{\theta\left(p_0\right)}{\mathrm{e}^{(p_0-\mu)/T}+1} + \frac{\theta\left(-p_0\right)}{\mathrm{e}^{-(p_0-\mu)/T}+1}\right], \tag{65}$$

with the following notations adopted in this Section: T is the temperature and μ is the chemical potential. With the use of the above formulas we obtain the following result for the contribution of the effects of finite temperature and density to the real part of the fermion energy shift:

$$\Delta\left(E^2\right) = 2e^2\int\frac{d^3k}{\left(2\pi\right)^3}\left[m^2 + m\theta + \frac{pk}{k^2}\left(k^2 - pk\right)\right]\times$$
$$\times\left\{\frac{\delta\left(k^2-2pk\right)}{k^2-\theta^2}\left[\frac{\theta\left(q_0\right)}{\mathrm{e}^{(q_0-\mu)/T}+1} + \frac{\theta\left(-q_0\right)}{\mathrm{e}^{-(-q_0-\mu)/T}+1}\right] - \frac{\delta\left(k^2-\theta^2\right)}{k^2-2pk}\frac{1}{\mathrm{e}^{|k_0|/T}-1}\right\}, \tag{66}$$

where $q_\mu = p_\mu - k_\mu$. The term with the step-function $\theta(q_0)$ accounts for the contribution of the electron gas and the term with $\theta(q_0)$ represents the contribution of the positron gas. Upon integration with the help of $\delta-$functions, leaving only the electron contribution, we obtain:

$$\Delta\left(E^2\right) = \frac{e^2}{8\pi}\int\frac{d\varepsilon}{\mathrm{e}^{(\varepsilon-\mu)/T}+1}\left(1 - \frac{2m^2+2m\theta+\theta^2/2}{\sqrt{\left(p_0\varepsilon+\theta^2/2-m^2\right)^2-\left(pq\right)^2}}\right), \tag{67}$$

where $\varepsilon = \sqrt{\mathbf{q}^2 + m^2}$. Hence, the following expression for the completely degenerate electron gas is obtained:

$$\Delta\left(E^2\right) = \frac{me^2}{8\pi}\left[\left(\frac{\mu}{m} - 1\right) - 2\left(1+\Theta\right)^2 \ln\left|\frac{\sqrt{\left(\frac{\mu}{m}\right)^2 + b\frac{\mu}{m} + c} + \frac{\mu}{m} + \frac{1}{2}b}{\sqrt{1+b+c} + 1 + \frac{1}{2}b}\right|\right], \tag{68}$$

where

$$\Theta = \frac{\theta}{2m}, \quad E_p = \sqrt{1 + \frac{\mathbf{p}^2}{m^2}}, \quad b = 2E_p\left(2\Theta^2 - 1\right), \quad c = E_p^2 + 4\Theta^4 - 4\Theta^2. \tag{69}$$

In massless (2+1)-QED, when $m \to 0$, with the finite Chern-Simons term, the dispersion law is as follows

$$E^2 = p^2 + \frac{e^2}{8\pi}\left[\mu - \frac{\theta}{2\pi}\left(\sqrt{4p\mu + \theta^2} - \theta\right)\right]. \tag{70}$$

In order to calculate the effective electron mass in the charge symmetric case at finite temperature, we should sum up series of the following form:

$$S\left(q,y\right) = \sum_{n=1}^{\infty} q^n \, \mathrm{e}^{ny}\, \mathrm{Ei}\left[-ny\right], \tag{71}$$

where $|q| < 1$, $\mathrm{Ei}[-x]$ is the integral exponential function and the parameter y is proportional to the inverse temperature T^{-1}. Introducing the polilogarithm function $\mathrm{Li}_s[q]$, we obtain with the help of the method developed in [64], [34]

$$S\left(q,y\right) = \int\limits_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i}\frac{\pi\Gamma\left(s\right)}{\sin\left(\pi s\right)} y^{-s}\mathrm{Li}\left[q\right], \quad 0 \ll c \ll 1 \tag{72}$$

where

$$\mathrm{Li}_s\left[q\right] = q\Phi\left[q,s,1\right] = \sum_{n=1}^{\infty}\frac{q^n}{n^s}. \tag{73}$$

Then closing the integration contour to the right (with $y \gg 1$) or to the left (with $y \ll 1$), we obtain the representation of $S(q,y)$ in the form of rapidly converging series of residues of the integrand at the points $s = 1, 2, 3, \ldots$ and $s = 0, -1, -2, -3, \ldots$ respectively. Thus for the mass shift of the electron at rest we finally obtain

$$\begin{aligned}\Delta\left(m^2\right) = \frac{e^2}{4\pi}\Bigg\{&-T\,\mathrm{Li}_1\left[-\,\mathrm{e}^{-m/T}\right] + \frac{\left(\theta+2m\right)^2}{4m}\Big(S\left[-\,\mathrm{e}^{-m/T}, y_1\right] - S\left[-\,\mathrm{e}^{-m/T}, y_2\right]\Big) + \\ &+ T\,\mathrm{Li}_1\left[\mathrm{e}^{-\theta/T}\right] - \frac{\left(\theta+2m\right)^2}{4m}\Big(S\left[\mathrm{e}^{-\theta/T}, y_3\right] - S\left[\mathrm{e}^{-\theta/T}, y_4\right]\Big)\Bigg\},\end{aligned} \tag{74}$$

where parameters y_i $(i = 1,2,3,4)$ are defined as

$$y_1 = \frac{\theta^2}{2mT}, \quad y_2 = \frac{1}{T}\left(2m - \frac{\theta^2}{2m}\right), \quad y_3 = \frac{1}{T}\left(\theta + \frac{\theta^2}{2m}\right), \quad y_4 = \frac{1}{T}\left(\theta - \frac{\theta^2}{2m}\right). \tag{75}$$

Let us discuss now two interesting results obtained in [33, 34, 65], where the finite temperature and density contributions to the radiative shift of the electron energy in the topologically massive (2+1)-QED were calculated.

Expressions (73), (74) yield the principal logarithmic approximation for the electron mass shift in the limiting case of high temperature. In the charge symmetric case, the one-loop radiative shift of the electron mass has the following asymptotic behavior [33, 34]:

$$\delta m = \begin{cases} \dfrac{e^2}{8\pi}\left(\ln\left(\dfrac{\theta}{2m}\right) + \dfrac{T}{m}\left(\ln(2) - 1\right) - \dfrac{T}{m}\ln\left(\dfrac{\theta}{T}\right)\right), \ \theta \ll 2m \ll T, \\ \dfrac{e^2}{2\pi}\left(\dfrac{T}{\theta}\ln\left(1 - \mathrm{e}^{-\theta/T}\right) - \dfrac{T}{\theta}\ln\left(1 + \mathrm{e}^{-m/T}\right)\right), \ 2m \ll \theta, \ 2mT \ll \theta^2. \end{cases} \tag{76}$$

This result at low temperatures ($T \ll \dfrac{\theta^2}{2m}$, $2m \ll \theta$) admits a limiting transition to massless electrodynamics [33, 34]:

$$\delta m|_{m\to 0} = \frac{e^2}{2\pi}\frac{T}{\theta}\left[\ln\left(1 - \mathrm{e}^{-\theta/T}\right) - \ln(2)\right]. \tag{77}$$

It follows from (77) that, at finite temperature in the initially massless (2+1)-QED with the Chern–Simons term, there exists an effect of the fermion mass generation in virtue of the radiative effects.

As follows from (76), (77), the temperature mass shift at low temperatures is negative. However, the total radiative electron mass shift also includes a part which is independent of temperature and is determined by the Chern-Simons term [4],[34]:

$$\Delta m_{\text{total}} = \Delta m\,(T=0) + \Delta m\,(T \neq 0) = \frac{e^2}{2\pi} + \Delta m\,(T). \tag{78}$$

Under the conditions (76), we have

$$\left|\frac{\Delta m\,(T)}{\Delta m\,(T=0)}\right| \propto \frac{T}{\theta} \ll 1, \quad T \ll \theta, \tag{79}$$

i.e. the temperature correction to the electron mass is small as compared to the $\Delta m(T=0)$ at $T \ll \theta$ and the total radiative electron mass shift remains positive. In the case of high temperature (76), we obtain qualitatively different behavior of the temperature mass shift:

$$\left|\frac{\Delta m\,(T)}{\Delta m\,(T=0)}\right| \propto \frac{T}{m}\ln\left(\frac{T}{\theta}\right) \gg 1, \quad T \gg 2m \gg \theta. \tag{80}$$

Thus, with growing temperature, the mass shift $\Delta m(T)$ changes its sign and its contribution to the radiative electron mass shift becomes dominant.

Finally, let us come back to the dispersion law and calculate the exchange correction to the thermodynamic potential:

$$\frac{\Omega_{\text{exch}}}{V} = \int \frac{d^3p}{(2\pi)^3}\frac{\Delta\left(E^2\right)}{2\sqrt{\mathbf{p}^2 + m^2}}\left(\mathrm{e}^{\left(\frac{E_p-\mu}{T}\right)} + 1\right)^{-1}. \tag{81}$$

In the case with $m = 0$, $T = 0$, i.e. massless (2+1)-QED at zero temperature, the dispersion law is given by (70) and the integration immediately results in

$$\frac{\Omega_{\rm exch}}{V} = \frac{e^2}{32\pi^2}\left\{\mu^2 - \frac{\theta}{2}\left[2\left(\sqrt{4\mu^2+\theta^2}-\theta\right) + \theta\ln\left(\frac{\sqrt{4\mu^2+\theta^2}-\theta}{\sqrt{4\mu^2+\theta^2}+\theta}\right) - 2\theta\ln\left(\frac{\mu}{\theta}\right)\right]\right\}. \quad (82)$$

It should be noted that whereas the one-loop mass operator in (2+1)-QED without the Chern-Simons term is infrared divergent on the mass shell, so that charge screening effects should be taken into account [66, 34], the mass operator in topologically massive QED at finite temperature and density is finite already in the one-loop approximation. Moreover, when $\theta \to 0$, the dispersion law (70) becomes

$$E^2 = p^2 + \frac{e^2\mu}{8\pi} \quad , \quad (83)$$

i.e. the finite density effects produce a gap in the spectrum. However, at any finite value of θ, the dispersion law looks as

$$E^2\,(p, \theta \neq 0) \xrightarrow{p\to 0} 0 \quad (84)$$

and the gap is not produced at least in the one-loop approximation. Thus, with the mass parameter θ tending to zero, there appears a gap in the electron energy spectrum that is due to finite density effects at zero temperature. However any finite value of this parameter prevents formation of a gap in the energy spectrum [34].

5 Induced parity-violating thermal effective action

As it was pointed out in the Introduction, a fundamental property of the gauge field action in odd-dimensional space-times is its non-invariance under "large" (homotopically non-trivial) gauge transformations. Hence, the expression $\exp(iS_{cs})$ will have a unique value only if a quantization condition is imposed on the Chern–Simons coefficient (the topological mass). This condition leads to quantization of the coefficient in units of $g^2/(4\pi)$, which results in restoration of invariance of the theory as a whole under large gauge transformation. The above conclusions are valid only for zero temperature quantum field theory. The situation seriously changes when temperature becomes finite. This is due to the fact that, at finite temperature, the fermion radiative corrections shift the tree level CS coefficient and become unquantized continuous functions of temperature [44, 67, 68]. Hence, at first glance, the problem of large gauge symmetry restoration seems to have no solution. However, in a number of recent publications [46, 47, 48, 69], it was demonstrated that, although the temperature dependent Chern–Simons coefficient may violate the invariance, there exist the other terms in the action which compensate this violation, so that the total effective action restores its gauge invariance. It turned out that the effective action can be constructed in such a way that it becomes invariant under both small and large gauge transformations at any temperature. This becomes possible by the use of a suitable regularization of the Dirac operator determinant, at the price of emerging parity anomalies.

As for the perturbative approach, it appears impossible to evaluate the effective action in a closed form, contrary to the (0+1) model, and this can be done only for a restricted class

of backgrounds, for example, for static fields, when $A_0 = A_0(t)$, $\mathbf{A} = \mathbf{A}(\mathbf{x})$ [70]. In the static limit, ($\mathbf{p} \to 0$, $p_0 = 0$), with all energies vanishing, at any order of perturbation, the "large" gauge invariance is not manifest. However, it is possible to sum up the leading order terms (this can be done through derivation of the Ward identity for the special case) in the parity violating effective action in this limit and the resulting effective action appears to have a form, which exactly coincides with the result obtained in the frame of special background gauge mentioned above. Moreover, this action is a generalization of the (0+1)-dimensional result and is invariant under large gauge transformations [70]. Therefore, the significance of calculation of the effective action in the special background is that it represents the leading term in the effective action in the static limit.

In [71], with the use of the technique developed in [51], an exact expression for this one-loop effective action was obtained, when the background field configuration was taken as a superposition of an abelian and a non-abelian gauge fields in the group $U(1) \times SU(2)$. Let us discuss the results of [71] in more detail.

5.1 Problem statement

The parity breaking part of the action can be written in the form

$$\Gamma_{odd}(A, M) = \frac{1}{2}\Big(\Gamma(A, M) - \Gamma(A, -M)\Big), \tag{85}$$

where the effective action $\Gamma(A, M)$ is related to the action for massive fermions $S_F(A, M)$ in the standard way

$$\exp\Big(-\Gamma(A, M)\Big) = \int D\psi D\overline{\psi} \exp\Big(-S_F(A, M)\Big).$$

The background field is the following combination of constant abelian ($U(1)$) and non-abelian ($SU(2)$) fields: $A_\mu = gA_\mu^{(1)a}T_a + eA_\mu^{(2)}I$. Here T_a are the $SU(2)$ group generators, and I is the unit matrix in the color space. Moreover, the following algebra of γ-matrices is used

$$\gamma^3 = \sigma^3, \quad \gamma^1 = i\sigma^1, \quad \gamma^2 = i\sigma^2, \quad \gamma^\mu\gamma^\nu = g^{\mu\nu} - i\epsilon^{\mu\nu\alpha}\gamma_\alpha.$$

Index 3 is related to the Euclidean time coordinate τ. Expression (85) can not be calculated exactly for the case of arbitrary fields A, and hence, we shall restrict ourselves to solving the problem in the above mentioned special gauge

$$\begin{aligned} &A_3^{(1),a} = |A_3^{(1)}(\tau)|n^a, \quad A_3^{(2)} = A_3^{(2)}(\tau), \\ &A_j^{(1)} = A_j^{(1)}(\mathbf{x}), \quad A_j^{(2)} = A_j^{(2)}(\mathbf{x}) \quad (j = 1, 2). \end{aligned} \tag{86}$$

Here, n^a is a fixed unit vector in the color space ($n^a n^a = 1$), and we also require that

$$[A_j, A_3] = 0, \quad [A_j, n] = 0. \tag{87}$$

Then, the effective action for massive fermions in the background gauge field (86) in the (2+1)-dimensional space-time at finite temperature can be written in the form

$$S_F(A, M) = \int_0^\beta d\tau \int d^2x\, \overline{\psi}(\gamma\partial + i\gamma A + M)\psi, \tag{88}$$

where $\gamma\partial = \gamma^\mu\partial_\mu$, $\gamma A = \gamma^\mu A_\mu$, and $\beta = 1/T$. Fermion and gauge fields that appear in (88) satisfy anti-periodic and periodic conditions, respectively

$$\psi(\beta, \mathbf{x}) = -\psi(0, \mathbf{x}), \quad \overline{\psi}(\beta, \mathbf{x}) = -\overline{\psi}(0, \mathbf{x}),$$

$$A_\mu(\beta, \mathbf{x}) = A_\mu(0, \mathbf{x}).$$

5.2 Parity breaking action

The fermion determinant can be written in the following form

$$\det(\gamma\partial + i\gamma A + M) = \int D\psi D\overline{\psi} \exp\Big(- \int_0^\beta d\tau \int d^2x \overline{\psi}(\gamma\partial + i\gamma A + M)\psi\Big). \tag{89}$$

Then, we perform the gauge transformation of fermion fields

$$\psi(\tau, \mathbf{x}) = \exp\Big(- i\Big[g\Omega^{(1)}(\tau)n + e\Omega^{(2)}(\tau)I\Big]\Big)\psi'(\tau, \mathbf{x}),$$

$$\overline{\psi}(\tau, \mathbf{x}) = \overline{\psi}\,'(\tau, \mathbf{x}) \exp\Big(i\Big[g\Omega^{(1)}(\tau)n + e\Omega^{(2)}(\tau)I\Big]\Big).$$

Since only the third components A_3 depend on τ in the gauge field configuration in question, and A_j are independent of τ, and also by virtue of condition (87), the transformations of this kind do not act on spatial components of the potential, and time dependence of the third component can be excluded, if we choose $\Omega^{(1)}$ and $\Omega^{(2)}$ in the form

$$\Omega^{(1)}(\tau) = -\int_0^\tau d\tau' A_3^{(1,n)}(\tau') + \Big(\frac{1}{\beta}\int_0^\beta d\tau' A_3^{(1,n)}(\tau')\Big)\tau,$$

$$\Omega^{(2)}(\tau) = -\int_0^\tau d\tau' A_3^{(2)}(\tau') + \Big(\frac{1}{\beta}\int_0^\beta d\tau' A_3^{(2)}(\tau')\Big)\tau.$$

Thus, the fermion determinant (89) takes the form

$$\det(\gamma\partial + i\gamma A + M) = \int D\psi D\overline{\psi} \exp\Big(- S_F(A_j, \tilde{A}_3, M)\Big), \tag{90}$$

where the action is equal to

$$S_F(A_j, \tilde{A}_3, M) == \int_0^\beta d\tau \int d^2x\, \overline{\psi}\Big(\gamma\partial + i(\gamma_j A_j + \gamma_3\tilde{A}_3) + M\Big)\psi, \tag{91}$$

and the quantity $\tilde{A}_3 = \frac{1}{\beta}\int_0^\beta d\tau\Big(gA_3^{(1)}(\tau)n + eA_3^{(2)}(\tau)I\Big)$ takes a constant value now. To calculate the determinant (90)– (91), perform Fourier-expansion of the fermion fields

$$\psi(\tau, \mathbf{x}) = \frac{1}{\beta}\sum_{n=-\infty}^{n=+\infty} e^{i\omega_n\tau}\psi_n(\mathbf{x}),$$

$$\overline{\psi}(\tau,\mathbf{x}) = \frac{1}{\beta}\sum_{n=-\infty}^{n=+\infty} e^{-i\omega_n\tau}\overline{\psi}_n(\mathbf{x}) \tag{92}$$

and then write the action as

$$S_F(A_j,\tilde{A}_3,M) == \frac{1}{\beta}\sum_{n=-\infty}^{n=+\infty}\int d^2x\,\overline{\psi}(\mathbf{x})_n\Big(\gamma d + i\gamma_3(\omega_n{+}\tilde{A}_3) + M\Big)\psi_n(\mathbf{x}).$$

Here, we introduced the following notation for the differentiation operator $\gamma d = \gamma_j(\partial_j + iA_j)$, and $\omega_n = \pi(2n+1)/\beta$ are Matsubara frequencies for fermions. Thus, taking into account that the fermion measure can now be written as

$$D\psi(\tau,x)D\overline{\psi}(\tau,x) = \prod_{n=-\infty}^{n=+\infty} D\psi_n(\mathbf{x})D\overline{\psi}_n(\mathbf{x}),$$

we obtain for the fermion determinant

$$\det(\gamma\partial + i\gamma A + M) = \prod_{n=-\infty}^{n=+\infty}\det\Big(\gamma d + M + i\gamma_3(\omega_n + \tilde{A}_3)\Big),$$

where the determinant behind the product sign can be written in the form

$$\int D\chi_n D\overline{\chi}_n \exp\Big(-\int d^2x\,\overline{\chi}_n(\mathbf{x})(\gamma d + \rho_n e^{i\gamma_3\phi_n})\chi_n(\mathbf{x})\Big). \tag{93}$$

Here, we made use of the Euler formula with the notations $\rho_n = \sqrt{M^2 + (\omega_n{+}\tilde{A}_3)^2}$ and $\phi_n = \arctan\Big(\dfrac{\omega_n + \tilde{A}_3}{M}\Big)$. To calculate determinant (93), the known method of calculating the anomalous Fujikawa Jacobian [72] can be used. To this end, we first make the following transformation of spinors χ (chiral rotation in the space $(1+1)$)

$$\chi_n(x) = \exp\Big(-i\frac{\phi_n}{2}\gamma_3\Big)\chi'_n(x),\ \overline{\chi}_n(x) = \overline{\chi}'_n(x)\exp\Big(i\frac{\phi_n}{2}\gamma_3\Big).$$

Then, it is easily verified that expression (93) takes the form

$$\det\Big(\gamma d + M + i\gamma_3(\omega_n + \tilde{A}_3)\Big) = J_n\det(\gamma d + \rho_n), \tag{94}$$

where

$$J_n = \exp\left\{-\frac{i}{4\pi}\mathrm{tr}\left(\phi_n\int d^2x\epsilon_{ij}\Big(F^{(1)}_{ij} + \frac{1}{2}F^{(2)}_{ij}\Big)\right)\right\}.$$

In this expression, $F^{(1)}$ and $F^{(2)}$ are the non-abelian and abelian field tensors, respectively. The last factor in (94) does not depend explicitly on the sign of the fermion mass and hence does not contribute to the parity breaking part of the effective action. Therefore, the expression for Γ_{odd} can be written right away

$$\Gamma_{odd} = \frac{i}{4\pi}\,\mathrm{tr}\left(\Big(\sum_{n=-\infty}^{n=+\infty}\phi_n\Big)\int d^2x\epsilon_{ij}\Big(F^{(1)}_{ij} + \frac{1}{2}F^{(2)}_{ij}\Big)\right). \tag{95}$$

Moreover, since the field ϕ_n itself should be expanded in color space directions $\phi_n = \phi_n^0 I + \phi_n^a T_a$, then with its explicit expression known, expression for its every color component can be easily obtained

$$\phi_n^0 = \frac{1}{2}\arctan\left(\frac{2M(\omega_n + e\tilde{A}_3^{(2)})}{M^2 + \frac{g^2}{4}|\tilde{A}_3^{(1)}|^2 - (\omega_n + e\tilde{A}_3^{(2)})^2}\right),$$

$$\phi_n^a = \arctan\left(\frac{gM|\tilde{A}_3^{(1)}|}{M^2 - \frac{g^2}{4}|\tilde{A}_3^{(1)}|^2 + (\omega_n + e\tilde{A}_3^{(2)})^2}\right) n^a.$$

Here, it should be emphasized that, for the combination of an abelian and non-abelian fields in question, unlike the case of a pure non-abelian field, not only the color component of the field ϕ_n^a but also component ϕ_n^0 will contribute to Γ_{odd}. This is related to the fact that the abelian field tensor $F^{(2)}$ is present in the integrand of the expression (95). In other words, the expression behind tr in (95) can be rewritten as

$$\int d^2x\epsilon_{ij}\left(\Big(\sum_{n=-\infty}^{n=+\infty}\phi_n^a\Big)F_{ij}^{(1)a} + \Big(\sum_{n=-\infty}^{n=+\infty}\phi_n^0\Big)\frac{1}{2}F_{ij}^{(2)}\right). \tag{96}$$

Now, consider calculation of one of the sums in (96), for instance $\sum\phi_n^a$, in more detail. To this end, introduce the following notations $m = \beta M$, $x = \frac{g}{2}\beta|\tilde{A}_3^{(1)}|$ and $y = e\beta|\tilde{A}_3^{(2)}|$. Then,

$$\sum_{n=-\infty}^{n=+\infty}\phi_n^a = \sum_{n=-\infty}^{n=+\infty}\arctan\left(\frac{2mx}{m^2 - x^2 + \Big((2n+1)\pi + y\Big)^2}\right).$$

Next, we use the equivalent form of this expression [51]

$$\sum(x, y, m) = \int_0^x du\frac{\partial\Sigma}{\partial u}(u, y, m),$$

where

$$\frac{\partial\Sigma}{\partial u}(u, y, m) = 2m\sum_{n=-\infty}^{n=+\infty}\frac{m^2 + u^2 + ((2n+1)\pi + y)^2}{[m^2 + ((2n+1)\pi + y)^2 - u^2]^2 + 4m^2u^2}.$$

This expression is exactly equal to

$$\frac{\partial\Sigma}{\partial u}(u, y, m) = = -\frac{m}{2\pi i}\oint_C dz\tanh\left(\frac{z}{2}\right)\frac{m^2 + u^2 + (y - iz)^2}{[m^2 + (y - iz)^2 - u^2]^2 + 4m^2u^2}, \tag{97}$$

where contour C encloses all the poles of $\tanh\left(\frac{z}{2}\right)$, i. e. points $z = i(2n+1)\pi$. Next, contour C should be replaced by the equivalent sum $C_1 + C_2$ of two other contours, which will together

enclose only four singular points of the fraction. Thus, summing up residues at all these points, we obtain the required expression (97)

$$\frac{\partial \Sigma}{\partial u}(u,y,m) = \frac{\sinh(m)}{4}\left[\frac{1}{\cosh(m)+\cos(u-y)} + \frac{1}{\cosh(m)+\cos(u+y)}\right] \tag{98}$$

and so, upon integration over u [73], the expression for the sum $\sum \phi_n^a$ takes the form

$$\sum_{n=-\infty}^{n=+\infty} \phi_n^a = \arctan\left(\tanh\left(\frac{m}{2}\right)\tan\left(\frac{x-y}{2}\right)\right) + \arctan\left(\tanh\left(\frac{m}{2}\right)\tan\left(\frac{x+y}{2}\right)\right). \tag{99}$$

It is clear that the sum $\sum \phi_n^0$ can be calculated in a similar way. Combining them, the final form of the expression (95) can be written as follows

$$\Gamma_{\text{odd}} = \Gamma^{(1)} + \Gamma^{(2)}, \tag{100}$$

where

$$\Gamma^{(1)} = \frac{ig}{8\pi}\Big(I_1 + I_2\Big)n^a \int d^2\, x\epsilon_{ij}F_{ij}^{(1)a}, \tag{101}$$

$$\Gamma^{(2)} = \frac{ie}{8\pi}\Big(I_1 - I_2\Big)\int d^2x\, \epsilon_{ij}F_{ij}^{(2)} \tag{102}$$

and

$$I_{1,2} = \arctan\left(\tanh\left(\frac{\beta M}{2}\right)\tan\left(\frac{g\beta}{4}|\tilde{A}_3^{(1)}| \pm \frac{e\beta}{2}|\tilde{A}_3^{(2)}|\right)\right).$$

It is easily seen that, in the limiting case, when one of the fields (either abelian or non-abelian) vanishes, the obtained expression exactly renders the earlier results [51]. One can also easily observe that, in the zero temperature limit, the parity breaking part of the action (100), (101), (102) goes over into the half-sum of the Chern-Simons terms for abelian and non-abelian fields, each of which reproduces the known results of the zero temperature quantum field theory [74, 75]

$$\Gamma_{\text{odd}}|_{T=0} = \frac{1}{2}\frac{M}{|M|}\Big(S_{CS}^{(1)} + S_{CS}^{(2)}\Big),$$

where in our case

$$S_{CS}^{(1)} = \frac{ig^2}{4\pi}\,\mathrm{tr}\int d^3x\, A_3^{(1)}\epsilon_{ij}F_{ij}^{(1)},$$
$$S_{CS}^{(2)} = \frac{ie^2}{4\pi}\int d^3x\, A_3^{(2)}\epsilon_{ij}F_{ij}^{(2)}.$$

6 Conclusions

In the present paper, we reviewed some results of investigations of radiative effects in the (2+1)-dimensional quantum electrodynamics and Yang-Mills theory with consideration for the influence of external fields, finite temperature and matter density.

In particular, a possibility of the Chern-Simons term generation due to non-zero external field as well as finite temperature and density in the (2+1)-dimensional QED was demonstrated. The radiative shift of the photon topological mass in (2+1)-QED in an external magnetic field as a function of the field strength and photon energy was analyzed. The calculation and analysis of the rate of the electron-positron pair photoproduction in (2+1)-QED in an external magnetic field was demonstrated and the probability of the process was presented. The electron mass shift in (2+1)-QED in an external magnetic field was calculated as a function of the electron energy and the fields strength. The Chern-Simons term contribution was also investigated. The calculation of the effective magnetic sensitivity of the 2D electron gas was performed, and the Chern-Simons term was shown to eliminate its divergence in weak external fields. The gap in the electron spectrum in (2+1)-QED was shown to appear due to the finite electron gas density effects. The topological term was shown to eliminate the gap and make the radiative mass shift converge in (2+1)-QED at finite temperature.

With the aim of obtaining the above results, detailed calculation of the photon polarization operator in (2+1)-QED in external magnetic field was performed. In particular, the case of relatively weak magnetic fields and high photon energies, $H \ll H_0$, $m \ll \omega$, was investigated. Comparison of the results, obtained for the photon elastic scattering in (2+1)-QED with the corresponding results in (3+1)-QED, demonstrated that the increase of the photon elastic scattering amplitude with growing parameter $\chi = H\omega/(H_0 m)$, when $\chi \gg 1$, in (2+1)-QED is determined by the factor $\chi^{1/3}$, whereas in (3+1)-QED it increases as $\chi^{2/3}$. In the opposite case, $\chi \ll 1$, the imaginary part of the scattering amplitude, responsible for the e^+e^- pair photo-production, behaves as $\chi^{1/2} \exp\left(-\dfrac{8}{3\chi}\right)$, whereas in (3+1)-QED, the corresponding pre-exponential is equal to χ. However, in the case $\chi \ll 1$, the real part of the photon scattering amplitude, which determines the photon mass $\Delta\left(m^2\right) = 2\omega \mathrm{Re}(T)$, is proportional to χ^2, and this coincides with the result of (3+1)-QED.

Calculation of the one-loop radiative energy shift of the ground state of an electron in a constant magnetic field in (2+1)-QED with the topological term was presented and the asymptotic behavior of the effective magnetic sensitivity $\chi_{2+1} = \partial\left(\Delta\varepsilon_0\right)/\partial\beta$ of the 2D electron gas was studied. Comparison of the obtained results for the magnetic sensitivity in the special case of topologically massless (2+1)-QED, $\theta = 0$, with those in (3+1)-QED reveals the following changes in the behavior of the magnetic susceptibility χ_{2+1} as a function of the field parameter $\beta = e_0 H/m^2$. In relatively weak fields, χ_{2+1} diverges logarithmically for $\beta \to 0$, whereas χ_{3+1} tends to a constant. In strong fields, χ_{2+1} decreases with growing field intensity faster than the corresponding function χ_{3+1} does. The consideration for the topological term $\theta \neq 0$ eliminates logarithmic divergence of the magnetic sensitivity χ_{2+1} in weak magnetic fields ($\beta \to 0$), i.e., $\chi_{2+1} \propto \ln\left(\theta/m\right)$. At finite temperature and density, consideration for the Chern-Simons term makes the electron mass operator finite already in the one-loop approximation, whereas it is infrared divergent on the mass shell in the one-loop approximation in the topologically massless (2+1)-QED.

We also presented the results of considering the influence of finite temperature and density on the radiative electron energy shift in the (2+1)-dimensional topologically massive QED. The case of a completely degenerate electron gas was studied. In the charge symmetric case, the electron effective mass at finite temperature was calculated and the limiting cases of low $T \ll \theta^2/(2m)$ and high $\theta \ll 2m \ll T$ temperatures were investigated. The mass shift due

to finite temperature, $\Delta m(T)$, was shown to be negative at low temperatures. However, it is small as compared to the zero temperature term $\Delta m(T=0)$, determined by the photon topological mass, so the total radiative electron mass shift remains positive. With growing temperature, the term $\Delta m(T)$ changes its sign and increases, so its contribution to the radiative electron mass shift becomes prevailing. Moreover, when $\theta \to 0$, the dispersion law becomes $E^2 = p^2 + e^2\mu/(8\pi)$, i.e. the finite density effects produce a gap in the electron spectrum. However, at any finite value of θ, the energy squared $E^2(p, \theta \neq 0)|_{p\to 0} \to 0$ and, at least in the one-loop approximation, the gap is no more present.

In addition to the above results, the problem of possible breaking of gauge invariance of the effective action due to the one-loop corrections to the CS coefficient in the (2+1)-dimensional (non-)abelian gauge theory at finite temperature was also considered in the present paper. For this purpose, the exact expression for the parity breaking part of the finite temperature (2+1)D effective action induced by massive fermions in the particular background, consisting of both abelian and non-abelian gauge fields, has been obtained. The special background configuration is characterized by the vanishing electric field and the time-independent magnetic field. In the limiting cases, when the abelian or non-abelian gauge field vanishes, the obtained general formula goes over into the earlier results obtained specifically for a non-abelian or an abelian fields respectively [51]. Likewise, in the limit of vanishing temperature, we received the zero temperature result obtained in earlier publications[74, 75]. It should be noticed, that our result explicitely demonstrates only the invariance under small gauge transformations that do not mix spatial and time components of the background fields. The large gauge invariance can be restored in the same way, as it was done in [45]–[48], i.e., through consideration of the full effective action, not restricted by the 1-loop approximation.

Thus, the following general conclusions can be made. The magnetic properties of photons change significantly with the reduction of the space-time dimensionality from (3+1) to (2+1). The radiative electron mass shift in a magnetic field and the effective magnetic sensitivity in the (2+1)-QED demonstrate that magnetic properties of electrons also change substantially, when the dimensionality is reduced from (3+1) to (2+1). Consideration for the finite topological photon mass $\theta \neq 0$ eliminates divergences in the electron mass operator and effective magnetic sensitivity, whereas finite temperature and density lead to new physical effects, such as appearance of the gap in the electron spectrum.

One of the general conclusions is also that large gauge invariance of the parity violating effective action at finite temperature in the (2+1)-dimensional space-time can not be obtained in the framework of the perturbative approach. This can only be achieved through consideration for contributions of all terms of all orders in the abelian or non-abelian gauge field action.

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